Addressing the Fleet Sizing Problem for Shared-and-Autonomous-Mobility Services

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1. Introduction

Shared mobility comes in many forms. Notable examples include carsharing services and ride-sourcing services (with dedicated drivers) services (Hartmans and Leskin, 2019; Jin et al., 2018; Loose, 2010; Shaheen and Cohen, 2019; Shaheen et al., 2018). Distinctive as they are from one another today, these shared mobility business models are expected to be consolidated into two major types of door-to-door mobility service in the foreseeable future due to the advent of autonomous vehicle technology, i.e., shared-and-autonomous-mobility service without pooling option (SAMw/oP) and the shared-and-autonomous-mobility service with pooling option (SAMw/P) (Stocker and Shaheen, 2017). This study aims to address a tactical fleet sizing problem in SAMw/P services by maximizing the profit of a service operator while taking the ride matching and vehicle dispatching (i.e., vehicle assignment and vehicle repositioning) into consideration. There has been research on SAM services, from system modeling and simulation to the fleet management and operational problems (Chen et al., 2016; Chen et al., 2020; Dandl et al., 2019; Fagnant and Kockelman, 2014; Fagnant et al., 2015; Hyland and Mahmassani, 2017; Vazifeh et al., 2018; Zhang et al., 2015; Zhao and Malikopoulos, 2019). The earliest studies considered the operation mode of traditional carsharing and ride-sourcing services without shared rides (Zhao and Malikopoulos, 2019). Recently, there have been some attempts to explore the option of carpooling in SAM services, i.e., the SAMw/oP services (Fagnant and Kockelman, 2018; Farhan and Chen, 2018; Gurumurthy et al., 2019; Hyland and Mahmassani, 2018; Ke et al., 2020; Levin et al., 2017). Nevertheless, most of these studies focused on impact analysis and simulation models with

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simple rule-based procedures or heuristic methods to pair rides and dispatch vehicles. To the best of our knowledge, no studies have ever considered the ride-matching problem, especially the joint optimization of ride matching and vehicle dispatching (RM&VD) for SAMFS problem using an exact optimization approach.

In this study, we consider a SAM service provider who dispatches a fleet of homogeneous shared autonomous vehicles (SAVs) to serve spatio-temporal customer ride requests within a service area. Each ride is characterized by its origin and destination, and a time window defined by the earliest departure time and the latest arrival time. We consider the most common carpooling scenario in the current ride-splitting services (e.g., UberPOOL and Didi Express Pool), i.e., "two riders-single vehicle". The way two riders are paired corresponds to a ride-matching pattern for the two riders. A ride-matching pattern is deemed feasible if it produces a positive cost saving in terms of driving distance while respecting the time window of each rider. The vehicle dispatching considered in this study includes vehicle assignment and vehicle repositioning. Given a limited fleet size, our objective is to maximize the daily profit of service providers by determining the optimal fleet size while considering the ride matching and vehicle dispatching strategy for the SAV fleet.

2. Methodology

We define the activity trajectories of vehicle as columns and formulate the SAMFS problem as a set partitioning model with side constraints. The huge number of feasible activity trajectories, however, makes the model intractable even for a small size problem. Luckily, the optimal solution could be found by a well-designed branch-and-cut-and-price (BCP) approach, a leading exact algorithm for solving many classes of VRP (Barnhart et al., 1998; Costa et al., 2019; Desaulniers et al., 2006; Feillet, 2010; Lübbecke and Desrosiers, 2005). When implementing the BCP method, the linear programming relaxation of the set partitioning model, referred to as master problem, is solved through a column generation method by repeatedly solving a restricted master problem with a subset of columns, and a pricing problem to find additional columns with positive reduced cost. The pricing problem embedded in the BCP approach to determine the RM&VD strategy of a vehicle is a variant of NP-hard elementary shortest path problem with resource constraint. To address the pricing problem, an effective two-phase method is developed to generate columns. Three speedup techniques are used to accelerate the algorithm. Valid inequalities are added to further strengthen the model and improve the upper bound. If the column generation method for solving the pricing problem produces a non-integer optimal solution, a branch-and-bound method is used to repeatedly solve the pricing problem until an integer solution is found. The proposed B&P approach can yield the optimal fleet size for SAMw/P services.

3. Results

Random instances are generated to evaluate the overall performance of the proposed BCP method and the effectiveness of the valid inequalities in obtaining the optimal integer solution in these instances. The results are shown in Table 1. It can be seen that most instances in the ordinary demand period can be solved to optimality within 2 hours if the number of rides is no larger than 40. Both the CPU time for solving the linear relaxation problem and that for obtaining the optimal integer solution would increase rapidly as the number of rides increases. Among the three demand scenarios, the instances in the ordinary demand period are the most computationally intensive, probably because of the long time horizon. On the contrary, the instances in the peak-hour demand period are the easiest to solve. In fact, all the instances label as 'p-XX-XX' in Table 1 are solved at the root node except one instance 'p-120-15'. The transition demand period lies in the middle. By comparing the results of these instances solved by the proposed approach with and without valid inequalities, we can find that the valid inequalities increase the likelihood of an instance being solved at the root node.

We further explore the effects of the number of rides and the slack time on the performance of SAMw/P services. The results are illustrated in Figure 1. We can see from Figure 1 (a) that with the increasing number of rides, the optimal fleet size increases steadily and somehow in a linear manner. On the contrary, the slack time has a negative effect on the fleet size, although the impact is marginal. The result is within our expectation because a larger slack time suggests that the riders are happy to accept a later arrival time at their destinations. This provides more flexibility to the service providers to employ an effective RM&VD plan which requires a smaller number of SAVs and achieves a high vehicle utilization. The findings are also consistent with the improved usage rate of SAV when the slack time becomes larger in Figure 1 (d). Figure 1 (b) illustrates the variation of profit in different scenarios. It shows that the profit will increase rapidly with more riders choosing the SAM services and the SAM services will become more profitable if the riders accept later arrival times. The results imply that the SAM service providers could earn more profit by serving more riders (see Figure 1 (c)) with a larger fleet (See Figure 1(a)). However, we caution that the increment of profit may decrease as the number of the rides grow further due to the law of diminishing marginal utility.

Table 1. Comparison of the results with and without valid inequalities for randomly generated instances

	Without valid inequalities							With valid inequalities							
Instance	#LPSolved	LP_CPU Time (s)	#Solved	#SolvedR	T_CPU Time (s)	#Column	#Node	#LPSolved	LP_CPU Time (s)	#Solved	#SolvedR	T_CPU Time (s)	#Column	#Cut	#Node
o-10-5	10	19.22	10	10	19.24	474	1.00	10	18.74	10	10	18.74	474	0	1.00
o-10-10	10	18.63	10	10	18.63	514	1.00	10	18.90	10	10	18.90	514	0	1.00
o-10-15	10	18.89	10	10	18.90	544	1.00	10	19.02	10	10	19.02	544	0	1.00
o-20-5	10	23.70	10	10	23.70	6,053	1.00	10	23.10	10	10	23.11	6,053	0	1.00
o-20-10	10	23.27	10	10	23.28	6,934	1.00	10	23.30	10	10	23.30	6,934	0	1.00
o-20-15	10	23.82	10	8	41.46	8,391	2.80	10	25.41	10	8	35.95	8,137	2.6	2.60
o-30-5	10	35.52	10	8	65.54	36,418	1.80	10	43.83	10	8	63.25	36,415	4	1.40
o-30-10	10	44.13	10	7	62.06	42,612	1.50	10	49.31	10	8	50.95	42,610	0.9	1.30
o-30-15	10	58.16	10	9	71.68	50,370	1.20	10	58.14	10	9	72.05	50,370	0	1.20
o-40-5	10	645.31	9	8	682.63	124,635	1.67	10	636.06	9	8	691.25	124,635	0	1.67
o-40-10	10	1084.78	10	7	1397.32	147,486	1.60	10	1081.10	10	7	1401.68	147,486	0	1.60
o-40-15	10	1795.60	9	9	1916.38	185,684	1.00	10	1787.77	9	9	1929.63	185,684	0	1.00
o-50-5	9	4470.92	5	5	2963.47	241,936	1.00	9	4517.93	5	5	2931.34	241,936	0	1.00
o-50-10	6	4795.78	3	3	5659.34	309,098	1.00	6	4823.22	3	3	5684.62	309,098	0	1.00
o-50-15	2	4447.72	0	0	-	-	-	2	4419.12	0	0	-	-	0	-
p-30-5	10	1.12	10	10	1.12	86	1.00	10	1.04	10	10	1.04	86	0	1.00
p-30-10	10	1.04	10	10	1.04	113	1.00	10	1.06	10	10	1.06	113	0	1.00
p-30-15	10	1.07	10	10	1.07	142	1.00	10	1.08	10	10	1.09	142	0	1.00
p-60-5	10	3.50	10	10	3.50	351	1.00	10	3.56	10	10	3.56	351	0	1.00
p-60-10	10	3.68	10	10	3.68	485	1.00	10	3.72	10	10	3.72	485	0	1.00
p-60-15	10	3.99	10	10	3.99	673	1.00	10	4.00	10	10	4.00	673	0	1.00
p-90-5	10	8.36	10	10	8.36	956	1.00	10	8.14	10	10	8.15	956	0	1.00
p-90-10	10	8.92	10	10	8.92	1,380	1.00	10	8.92	10	10	8.92	1,380	0	1.00
p-90-15	10	10.17	10	10	10.18	1,972	1.00	10	10.23	10	10	10.23	1,972	0	1.00
p-120-5	10	17.90	10	10	17.90	1,726	1.00	10	16.60	10	10	16.60	1,726	0	1.00
p-120-10	10	19.01	10	10	19.02	2,597	1.00	10	18.82	10	10	18.83	2,597	0	1.00
p-120-15	10	22.57	10	9	35.24	3,897	2.00	10	22.24	10	9	34.97	3,897	0	2.00
t-30-5	10	14.27	10	10	14.30	2,313	1.00	10	13.66	10	10	13.66	2,313	0	1.00
t-30-10	10	14.06	10	9	14.79	2,936	1.60	10	13.80	10	9	13.83	2,936	0	1.60
t-30-15	10	14.10	10	9	139.06	3,726	5.20	10	16.92	10	9	151.08	3,726	3.6	1.80
t-60-5	10	22.79	10	10	22.80	20,436	1.00	10	22.40	10	10	22.40	20,436	0	1.00
t-60-10	10	24.66	10	6	158.00	28,277	10.60	10	26.88	10	7	166.48	28,277	0.7	8.20
t-60-15	10	33.16	9	6	114.51	38,499	5.67	10	36.23	9	7	109.04	38,090	0.3	5.00
t-90-5	10	450.75	10	6	736.36	75,852	2.40	10	448.64	10	6	750.87	75,852	0	2.40
t-90-10	10	1298.69	7	6	1379.62	98,133	2.14	10	1300.44	7	6	1407.97	98,133	0	2.14
t-90-15	10	3086.52	2	2	1411.31	106,885	1.00	10	3089.12	2	2	1472.18	106,885	0	1.00
t-120-5	5	3522.32	2	2	3610.19	175,612	1.00	5	3567.10	2	2	3684.86	175,612	0	1.00
t-120-10	2	5131.57	0	0	-	-	-	2	5094.71	0	0	-	-	-	-
t-120-15	0	-	0	0	-	-	-	0	-	0	0	-	-	-	-

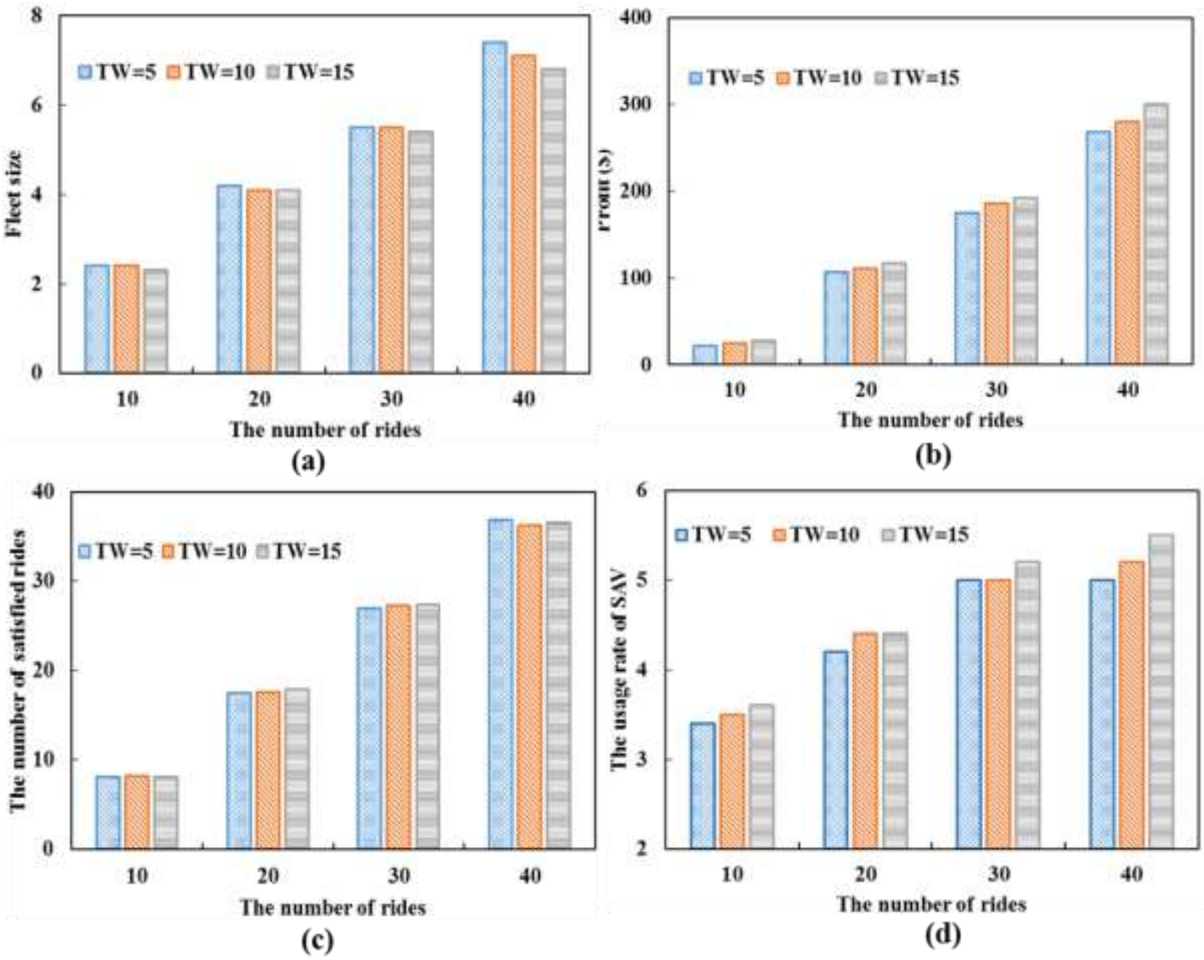

Figure 1. Effects of the ride number and the slack time on system performance

4. Conclusions

This study investigated the fleet sizing problem of SAM services considering ride-pooling and vehicle dispatching. A set parking model was formulated for the considered problem. A tailored BCP approach was developed to find a global optimal solution. The pricing problem within the BCP approach is NP-hard in the strong sense and we proposed a customized two-phase method to effectively address it. The solution methods were compared and evaluated by numerical experiments and the results have demonstrated their competence under different problem settings. Further research work can be undertaken in several aspects, among which the first and most important future work is to develop more efficient algorithms or heuristic methods for solving large-scale problems. The current sensitivity analyses were also limited by the instances that can be solved to optimality. In the future when more advanced and efficient methods are available, real-life case studies and impact analyses need to be conducted.

5. References

Barnhart, C., Johnson, E.L., Nemhauser, G.L., Savelsbergh, M.W., Vance, P.H., 1998. Branch-and-price: Column generation for solving huge integer programs. *Operations Research* 46(3), 316-329.

Chen, T.D., Kockelman, K.M., Hanna, J.P., 2016. Operations of a shared, autonomous, electric vehicle fleet: Implications of vehicle & charging infrastructure decisions. *Transportation Research Part A: Policy and Practice* 94, 243-254.

Chen, X. M., Zheng, H., Ke, J., Yang, H., 2020. Dynamic optimization strategies for on-demand ride services platform: Surge pricing, commission rate, and incentives. *Transportation Research Part B: Methodological*, *138*, 23-45.

Costa, L., Contardo, C., Desaulniers, G., 2019. Exact branch-price-and-cut algorithms for vehicle routing. *Transportation Science* 53(4), 946-985.

Dandl, F., Hyland, M., Bogenberger, K., Mahmassani, H.S., 2019. Evaluating the impact of spatio-temporal demand forecast aggregation on the operational performance of shared autonomous mobility fleets. *Transportation* 46(6), 1975-1996.

Desaulniers, G., Desrosiers, J., Solomon, M.M., 2006. *Column Generation*. Springer Science & Business Media.

Fagnant, D.J., Kockelman, K.M., 2014. The travel and environmental implications of shared autonomous vehicles, using agent-based model scenarios. *Transportation Research Part C: Emerging Technologies* 40, 1-13.

Fagnant, D.J., Kockelman, K.M., 2018. Dynamic ride-sharing and fleet sizing for a system of shared autonomous vehicles in Austin, Texas. *Transportation* 45(1), 143-158.

Fagnant, D.J., Kockelman, K.M., Bansal, P., 2015. Operations of shared autonomous vehicle fleet for Austin, Texas, market. *Transportation Research Record* 2563(1), 98-106.

Farhan, J., Chen, T.D., 2018. Impact of ridesharing on operational efficiency of shared autonomous electric vehicle fleet. *Proceedings of the 97th Annual Meeting of Transportation Research Board,* Washington, U.S. Jan 2018.

Feillet, D., 2010. A tutorial on column generation and branch-and-price for vehicle routing problems. *4OR* 8(4), 407-424.

Gurumurthy, K.M., Kockelman, K.M., Simoni, M.D., 2019. Benefits and costs of ride-sharing in shared automated vehicles across Austin, Texas: opportunities for congestion pricing. *Transportation Research Record* 2673(6), 548-556.

Hartmans, A., Leskin, P., 2019. The history of how Uber went from the most feared startup in the world to its massive IPO. *Business Insider*. https://www.businessinsider.com/ubers-history (accessed on 3 May 2020).

Hyland, M.F., Mahmassani, H.S., 2017. Taxonomy of shared autonomous vehicle fleet management problems to inform future transportation mobility. *Transportation Research Record* 2653(1), 26-34.

Hyland, M.F., Mahmassani, H.S., 2018. Sharing is caring: dynamic autonomous vehicle fleet operations under demand surges. *Proceedings of the 97th Annual Meeting of Transportation Research Board,* Washington, U.S. Jan 2018.

Jin, S.T., Kong, H., Wu, R., Sui, D.Z., 2018. Ridesourcing, the sharing economy, and the future of cities. *Cities* 76, 96-104.

Ke, J., Yang, H., Zheng, Z., 2020b. On ride-pooling and traffic congestion. *Transportation Research Part B: Methodological*, *142*, 213-231.

Levin, M.W., Kockelman, K.M., Boyles, S.D., Li, T., 2017. A general framework for modeling shared autonomous vehicles with dynamic network-loading and dynamic ride-sharing application. *Computers, Environment and Urban Systems* 64, 373-383.

Lübbecke, M.E., Desrosiers, J., 2005. Selected topics in column generation. *Operations Research* 53(6), 1007-1023.

Shaheen, S., Cohen, A., 2019. Shared ride services in North America: definitions, impacts, and the future of pooling. *Transport Reviews* 39(4), 427-442.

Shaheen, S., Cohen, A., Jaffee, M., 2018. Innovative mobility: Carsharing outlook. https://escholarship.org/content/qt1mw8n13h/qt1mw8n13h.pdf (accessed on 3 May 2020).

Stocker, A., Shaheen, S., 2017. Shared automated vehicles: Review of business models. International Transport Forum Discussion Paper. https://www.itf-oecd.org/shared-automated-vehicles-review-business-models (accessed 3 May 2020).

Vazifeh, M. M., Santi, P., Resta, G., Strogatz, S. H., Ratti, C., 2018. Addressing the minimum fleet problem in on-demand urban mobility. *Nature, 557*(7706), 534-538.

Zhang, W., Guhathakurta, S., Fang, J., Zhang, G., 2015. Exploring the impact of shared autonomous vehicles on urban parking demand: An agent-based simulation approach. *Sustainable Cities and Society* 19, 34-45.

Zhao, L., Malikopoulos, A.A., 2019. Enhanced mobility with connectivity and automation: A review of shared autonomous vehicle systems. *IEEE Intelligent Transportation Systems Magazine*.